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Introduction

The purpose of this paper is to analyse Pasinetti's notion of productive capacity in terms of vertically integrated units, which he in his **Metroeconomica** article (1973) and which underlay his growth model put forward in his "A New Theoretical Approach to the Economic Growth" (1965) and later on, more extensively, in his *Structural Change and Economic Growth* (1981) and more recently in his *Structural Economic Dynamics* (1983).

Since this notion forms the core of Pasinetti's argument, we shall limit our analysis to the concept of a 'unit of vertically integrated productive capacity' -K_i- and its relationship with the derived matrix H, which we might denote the matrix of the vertically integrated productive capacity sector.

Pasinetti's *unit* of vertically integrated productive capacity

We may begin our analysis by stating Pasinetti's well known definition of the units of vertically integrated productive capacities as the columns of matrix H:

$$H \equiv A(B - A)^{-1} = [H_{ij}]$$
$$h_j = A(B - A)^{-1} e_j \qquad (1)$$

where e_j is a i^{th} unit column vector refering to the j^{th} final commodity. Regarding this formulation Pasinetti writes:

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> "Each column vector h_i in (4.36) - (1 above) - expresses in a consolidated way the series of heterogeneous physical quantities of commodities 1, 2, ..., m, which are directly and indirectly required as stocks, in the whole economic system, in order to obtain one physical unit of commodity i as a final good (i = 1, 2, ..., m). This is another particular composite commodity which we shall call a unit of vertically integrated productive capacity for commodity i (i = 1, 2, ..., m)". (1973, p.6)

From this definition we have that each column vector h_j represents the 'physical unit of vertically integrated productive capacity' in terms of a *composite commodity*, the particularity of each such unit being given by the different proportions in which the heterogeneous physical commodities i - represented by the various entries in each column - are directly and indirectly used as capital stock in the whole economy for the production of a unit of final commodity j. In this way column vector h_j is also specific to the final commodity for whose production it is intended. Moreover, Pasinetti has introduced a column vector $K = [K_i]$ (i = 1, 2, ..., m) where each scalar K represents *conceptually*, a column vector h_j. Each K_i is then called the "stock of capital goods measured in terms of physical units of vertically integrated productive capacity" (1981, p.37).

The only justification Pasinetti adduces for consolidating the heterogeneous set of commodities making up each column vector h_j into a single unit - scalar K_j. - is based on the conception that any commodity is a composite commodity.[1] Thus, instead of describing a

1. Pasinetti writes: "We may go back to the physical quantity system. So far in the analysis all commodities have been measured in terms of the physical units that are commonly used to measure them (e.g. tons, bushels, numbers, etc.). But expressions (4.7)(4.36) suggest the possibility of an alternative physical unit of measurement for capital goods. More precisely they suggest the possibility of measuring capital goods in terms of a particular *composite* commodity, which we may call 'physical unit of vertically integrated productive capacity'. [...] For the purpose of our analysis, a composite commodity does not present any *conceptual* difficulty. (As a matter of fact, any commodity, e.g. a pair of shoes - can always be considered as composed of various elementary commodities - i.e. leather, string, rubber, etc. - put together in fixed proportions)." (1973, p.10. Emphasis added in both cases).

composite commodity by its components (as is done by the entries in each column vector h_j) we can introduce a unit of that set of produced commodities -whatever they are - that suffice to produce (with units of vertically integrated labour inputs) a unit of the corresponding final commodity.

Looked at in this way, this new concept of a ‘unit of productive capacity K_i’ seems to be a very simple, clearly defined and convenient concept. Nevertheless, on closer analysis it reveals some problems and ambiguities in its interpretation. On the one hand, if K_j is only a ‘conceptual’ representation of the column vector h_j, which is a derived vector $(A(B - A)^{-1} e_j)$, justified by saying that any commodity is a composite commodity, we might as well do without this shorthand form and carry on our analysis explicitly in terms of matrix H. On the other hand, if K_j is too ‘conceptually distant’ from h_j, then it might be devoid of any analytical meaning. These problems are further enhanced if we bear in mind that although matrix H is not a directly observable matrix, it is a derived matrix, derived from the directly observable transactions matrix, and therefore its entries are expressed in terms of the same set of heterogeneous physical commodities as is the Leontief input-output matrix, and thus the relationships shown in this H matrix will also break down when technical progress is present. In other words, matrix H, thought of as simply $A(B - A)^{-1}$, suffers from the same dynamic inconsistency that Pasinetti correctly attributes to the dynamic Leontief input-output model. It is at this point that the introduction of the K_i units becomes necessary, because by consolidating all the ordinary heterogeneous physical commodities into a single conceptual unit one frees the model from all these changes going on at the physical or observed level. Capital goods expressed in terms of these units of vertically integrated productive capacities acquire an autonomy of their own and their behaviour through time is independent of the continuous physical and qualitative changes that technical progress may bring about. Thus, when these units of productive capacity (K_i's) are used, *and only then*, the model's dynamic consistency is restored. It is quite clear that the relevance of these units is closely attached to the assumption of technical progress and structural change. In the absence of such changes there would be no need for these new concepts[2] and input-output models would give us all the information we need about the economy both at one point in time and through time.

Vertical integration and input-output analysis

Let us turn briefly to Pasinetti's own discussion of the analytical significance of the notions of vertical integration with respect to the input-output model.

In various places of Chapter VI of his book ("The Empirical Significance of Vertically Integrated Analysis"), Pasinetti writes:

> "In a vertically integrated model, the criterion is the process of production of a final commodity, and the problem is to build *conceptually* behind each final commodity a vertically integrated sector which, by passing through all the intermediate commodities, goes right back to the original inputs." (1981 p. 113, original emphasis)
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> "In this way, each vertically integrated sector is reduced to one flow input of labour and to one stock-quantity of capital goods; or, more specifically, to one vertically integrated labour coefficient and to one vertically integrated unit of productive capacity." (p. 114)
>
> "A vertically integrated sector is, therefore, from an inter-industry point of view, a very complex 'sector' as it repeatedly goes through the whole intricate pattern of inter-industry connections. However, from the point of view of the homogeneity of the inputs, it becomes a very simple one, as it eliminates all intermediate goods and resolves each final commodity into its *ultimate*

2. Of course, the notions and algebraic structure of vertical integration might still be very useful for distribution theory.

> *constituent elements*: a (flow) quantity of labour and a (stock) quantity of capital goods." (p. 110, emphasis added).

Various important issues come out explicitly from these statements. First, we can mention Pasinetti's view of a vertically integrated sector as a conceptual construct whose empirical link with the directly observable input-output system is the H matrix. It should be clear that the conceptual construction that is mostly needed is that to the specific *unit*, that is, the "one vertically integrated productive capacity" for all the other vertically integrated (labour) units can unambiguously be derived from the observed data. Secondly, and more interesting in terms of our analysis, is his claim that vertical integration, by consolidating or going over all the intermediate stages, resolves each final commodity into its "ultimate constituent elements" a unit of labour and a unit of capital goods (of course measured in terms of vertically integrated units).

We have called this second aspect of Pasinetti's statement interesting for its apparently contradictory character when confronted with another statement, where he says:

> "The theory of value implied by the present theoretical scheme becomes a theory in terms of simple labour - *a pure labour theory of value*." (1981, p. 132, Emphasis in the original)

And further on:

> "Labour emerges from the very logic of the present analysis as the *only ultimate factor of production*." (Ibid, p. 133. Emphasis added).

When a *unit* is not really a *unit*

To analyse these apparently contradictory statements, let us turn to the algebraic formulation of the model. For this purpose we will make use of the fixed capital goods in general model - i.e. joint production - and we will make the simplifying assumption that $(B - A)^{-1}$ is non-negative. In this case we may begin by writing the technical input and output matrices A and B as follows:

$$A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix} \tag{2 }$$

where the first n-1 rows and columns refer to the consumption goods sector, and the last n-1 columns to the capital goods sector. From these matrices, and the vector of direct labour inputs $a_{[n]} = [a_{01} \quad a_{02}]$ we get:

$$(B-A)^{-1} = \begin{bmatrix} (B_{11} - A_{11})^{-1} & 0 \\ -(B_{22}-A_{22})^{-1}\,(B_{21}-A_{21})(B_{11}-A_{11})^{-1} & (B_{22}-A_{22})^{-1} \end{bmatrix}$$

$$H \equiv A(B-A)^{-1} = \begin{bmatrix} A_{11}(B_{11}-A_{11})^{-1} & 0 \\ A_{21}(B_{11}-A_{11})^{-1} - A_{22}(B_{22}-A_{22})^{-1}(B_{21}-A_{21})(B_{11}-A_{11}) & A_{22}(B_{22}-A_{22})^{-1} \end{bmatrix} \tag{3}$$

$$a_{[n]}(B-A)_{-1} \equiv \upsilon = \begin{bmatrix} a_{01}(B_{11}-A_{11})^{-1} - a_{02}(B_{22}-A_{22})^{-1}B_{21} - A_{21}(B_{11}-A_{11})^{-}1 & a_{02}(B_{22}-A_{22})^{-1} \end{bmatrix}$$

Following Pasinetti, we can write the above system of equations, which represents the economy in terms of vertically integrated sectors (i.e. in terms of the columns of the derived matrix H and the scalars v_j (j = 1, 2, ..., n-1 - or m in Pasinetti's notation)) in terms of (vertically integrated) *units* of productive capacity and labour:

$$H = \begin{bmatrix} \hat{k}_{j1} & 0 \\ 0 & \hat{k}_{j2} \end{bmatrix}$$

$$\upsilon = [\upsilon_1 \quad \upsilon_2]$$

As a convention $\hat{k}_{j1}$ *and* $\hat{k}_{j2}$ are diagonal (sub)matrices.

At the beginning of this paper we said that Pasinetti defines each unit of productive capacity (vertically integrated) - $\hat{k}$ above - with reference to the final commodity for whose production it is intended. Therefore, in this way we must interpret $\hat{k}_{j1}$ as those units of productive capacity for the production of consumer goods as a final commodity (thus the second suffix 1), and the $\hat{k}_{j2}$ as those units of productive capacity for the production of capital goods as a final commodity. (Thus the second suffix 2). Moreover, we have seen that these notions become relevant for the analysis of a dynamic system, and we already know that for Pasinetti a central feature of a dynamic analysis of growth is that, in a modern industrial economy, technical progress brings about not only quantitative changes but also qualitative structural changes. Nevertheless, it is of great importance to note that technical progress in this framework is strictly limited to the production side of the theory, that is to the changes occurring in the means of production relative to the *same* and clearly defined final goods.[3]

This must be so, because since each unit of productive capacity is defined with respect to the final commodity for whose production it is intended; if this latter commodity changes so will the meaning of the former. It follows from this that in this framework, as in the Leontief input-output model, we are not able to deal with the problem of new-final-commodities. But, by contrast to it, this definition of technical progress, when viewed in the light of qualitative changes, does allow for the introduction of new commodities, as long as they are not used as final commodities but are only used for the production of the final commodities. In other words, although actually the columns of matrix H are derived from the observed data, from which we are supposed to build, conceptually, a unit of (vertically integrated) productive capacity, in theory what we are enabling the system to do is to drop some rows and columns of matrices A and B (in p. 5 above) - at least conceptually - and either leaving them out or replacing them with new ones, while keeping the definition of a final commodity intact. Note that technical progress can, as it is conceived to do, alter the physical composition of the means of production, not only by a mere change in quantities but by a physical and qualitative process of change. This is why the representation of matrix H in terms of the units of productive capacity - $\hat{k}_j$ - becomes not only relevant now, but

3. In fact, in Chapter IX, Section 18 ("The Meaning of Technical Progress") Pasinetti writes: "The term 'technical change' is a very general one. But we may now ask more specifically: when is it that technical 'change' actually means technical 'progress'? First of all, there obviously is 'progress' when technical change takes the form of invention of entirely new, previously unknown, goods. But we are concerned here with that type of technical change which consists of improvements in the production of already known goods." (1981, p. 206).

necessary; it enables us to disregard all the many and complex physical and qualitative changes that are taking place and that affect the definition of a physical capital good.

Produced means of production as weighted quantities of labour

As we mentioned earlier on, capital goods, when expressed in terms of vertically integrated units of productive capacity, are regarded as that set of produced commodities, whatever they are in terms of their composition - that together with labour are sufficient to produce a unit of a clearly defined and unchanging final commodity.[4] In terms of vertically integrated sectors the production of all final commodities has in fact been reduced to their 'ultimate' components: a (flow) quantity of labour and a (stock) quantity of capital.

Having said this the problem now seems to be the claim that labour is the only ultimate factor of production. If in terms of vertically integrated sectors we have decomposed all production activities into two ultimate components, but in terms of the logical construction of the model there is only one such ultimate component, there must be a missing link between vertical integration and the model's logical framework. This link can be found if we use the notions of vertical integration for the analysis of distribution and value.

We may begin such an analysis by formulating our original price equation for system (2) and equations (3) as follows:

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4. This treatment of a final commodity as an 'unchanging' and 'clearly defined' commodity might be best understood in terms of Kelvin Lancaster's 'characterization' of commodities where goods are defined by a set of objective characteristics, and individuals react to the different characteristics of the goods. (Lancaster, 1971).

$$pB = a_{oj}w + pA + pA\pi \tag{4}$$

From which after some manipulations and given (3) we get:

$$p = \upsilon w + pH\pi \tag{5}$$

This equation (5) corresponds to Pasinetti's claim that in terms of vertically integrated sectors (depicted here by the use of row vector υ and matrix H) all commodities are reduced to a single labour and capital input as their ultimate components. Nevertheless equation (5) is not a reduced solution for we can still write it as:

$$p = \upsilon w + [\upsilon w + pH\pi] H\pi \tag{6}$$

or more conveniently:

$$p = \upsilon w + \upsilon_2 w\pi + (pH\pi)^2 \quad \text{where} \quad \upsilon_2 = \upsilon H \tag{6a}$$

Furthermore we have:

$$p = \upsilon w + \upsilon_2 w\pi + [\upsilon w + pH\pi] (H\pi)^2 \tag{6b}$$

$$p = \upsilon w + \upsilon_2 w\pi + \upsilon_3 w\pi^2 + (pH\pi)^3 \quad \text{where} \quad \upsilon_3 = \upsilon_2 H = (\upsilon H) H$$

We could follow this process as an infinite series as the power (call it z) becomes infinite:

$$p = \upsilon w [I + H\pi + (H\pi)^2 + (H\pi)^3 + \ldots] \tag{7}$$

The series being convergent if $H\pi$ tends to zero (0) as the power becomes infinite. In this case the iterative method will converge to the inverse matrix $(I - \pi H)^{-1}$, and we can write the solution for equation (5) as:

$$p = \upsilon (I - \pi H)^{-1} w \tag{5a}$$

As we can now see, vertical integration and the process of "higher order vertical integration", as Pasinetti calls υ_z and H^z (z = 1, 2, 3, ...), have enabled him to reduce the system, in each round of approximations, to their ultimate logical component: labour.

The logic of this result is based on the meaning of the process of higher order vertical integration which stems from the fact that in our price equation (5) we can theoretically build a vertically integrated sector for the capital goods expressed in terms of vertically integrated units, i.e. we can re-write for the second addendum on the right hand side of equation (5):

$$P_K \equiv pH\pi = [\,\upsilon w + pH\pi\,]\,H\pi \qquad (8)$$

where P_K is the price of the 'unit of productive capacity' in vertically integrated *units*. Thus:

$$P_K = \upsilon\,(H\pi)w + P_K\,(H\pi) \qquad (9)$$

and, as we have seen, we can continue this process back to every infinite series of residuals P_K. In fact, what we are doing here is reducing prices to a series of weighted quantities of labour, and this logical procedure provides the basis for the apparently contradictory character of Pasinetti's statements we quoted in page 5 above.

Concluding Remarks

In the preceding pages we have analysed Pasinetti's notion of productive capacity in terms of the concept of a 'unit of vertically integrated productive capacity K_i' in relation to the derived matrix H, the matrix of the vertically integrated productive capacity sector.

We saw that these K_i units are supposed to represent, conceptually, the columns h_j of matrix H, but since this latter is expressed in terms of the ordinary physical units, it suffers from the same dynamic inconsistency that Pasinetti has correctly demonstrated for the Leontief's input-output model. Therefore these units K_i cannot be thought of as simply the composite representation of $h_j = A(B - A)^{-1} e_j$, their being a composite commodity must itself be freed of those physical commodities that make it up. That is, conceptually the K_i's must be thought of as a composite commodity - e.g. a shoe (as a factor of production) - but it cannot be defined as such only because it is made up of "leather, string, rubber, etc.", rather they must be thought of as a set of produced commodities, whatever they are, and however they change (in its dynamic dimension), that with labour suffice to produce a unit of the clearly defined final commodity. It is in these terms, *and in these terms only*, that we must regard the K_i units as a composite commodity; consolidating all the different ordinary heterogeneous physical commodities, whatever they are at any point in time that are used for the production of a given final commodity.

We saw, furthermore, that the above analysis was based on two fundamental principles: (i) that this notion of a unit of vertically integrated productive capacity was relevant for a dynamic analysis of technical progress and structural change, and (ii) that this notion of dynamic, in terms of qualitative changes affected only the nature of the means of production while leaving unchanged the definition of a final commodity. We mentioned that conceptually there is no problem with the clear definition of a final commodity even in the most dynamic conceptual framework, if we define a final commodity in terms of its properties or objective characteristics as suggested by Lancaster (1971).

Moreover we saw that these vertically integrated notions do indeed reduce and simplify the analysis of a dynamic economic system in terms of their ultimate components a unit of vertically integrated labour input and a unit of vertically integrated productive capacity. But the components must be regarded as ultimate only with respect to the algebraic formulation of a dynamic economic system expressed in terms of vertically integrated notions. This does not mean that at the very end there is, besides labour, something else called capital that enters into the production of some other goods. In this context capital goods are produced capital goods and as such are subject to the continuous process of vertical integration, or vertical hyper integration. Once we carry out this procedure we found that there is only one final or ultimate component of all produced commodities and that is labour.

Finally we must say although there is an apparently direct empirical link between the observed physical magnitudes and the conceptual ones (through the derived H matrix), this link cannot - in its dynamic dimension - but be a purely conceptual one for these K_i units must be completely independent of the physical composition of the columns of matrix H. This property is further enhanced by the working definition of technical progress that Pasinetti has assumed whereby the only commodities that are affected are those used as means of production, while the definition of the final goods remains unaffected. So what we are in fact doing is disregarding the changes occurring in the physical composition of the produced means of production, they are what they are in terms of commodities. But the logical justification for doing this, after all, seems to be that these produced means of production are nothing else but weighted quantities of labour, so we can obviate the physical aspect of these capital goods.

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