

*Modeling cycle and trend  
in macroeconomic time series:  
an application to Puerto Rico*

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## **Modeling cycle and trend in macroeconomic time series: an application to Puerto Rico**

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**Abstract** The aim of this study is to summarize the alternative methods developed to distinct between a deterministic trend and a stochastic trend. Meanwhile, it estimates the proportion of the permanent and the transitory components and the trend shifting model for real output in Puerto Rico.

Results pointed toward a significant fraction of the real GDP fluctuations in Puerto Rico can be attributed to the permanent component--between 14 to 60 percent. Likewise, empirical evidence for real output supports no mean-reversion but rather small and larger shocks, other than 1973-74 oil shock, have affected real output fluctuations. At last, real output exhibits a stochastic trend rather that a deterministic one.

### **1. Introduction**

Traditionally, many business cycles analysts have suggested that most macroeconomic time series can be decomposed into two main components; a transitory (cyclical or stationary) element and the trend (permanent) component. The first component refers to the temporary fluctuations associated mainly with the business cycle while the latter attempts to describe the long-run growth trend, commonly referred to as deterministic<sup>1</sup>. This trend is usually portrayed using a simple linear regression model.

In 1982 Nelson and Plosser in a path-breaking paper found evidence in direct contrast to the traditional view on business cycles of the United States. They found no

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1. A convenient way to understand transitory changes is, say, output fluctuations due to changes in labour or demand of labour, changes in money supply, interest rate, bad crops, etc. Conversely, a permanent movement can be understood as those motivated by changes in productivity due to technological changes, new oil or material prices regimes, modifications in the monetary system, etc. The recent approach to decomposition challenges the traditional view of treating the trend component in output as being smooth with short-run fluctuations coming from its transitory component.

evidence to reject the hypothesis that there is a stochastic trend in real and nominal output, wages, prices, monetary variables and asset prices, except for the rate of unemployment. Their analysis indicated that most of the innovation variance in these series should be allocated to the nonstationary (permanent) component, with little variance remaining for the stationary or cyclical component. Technically speaking, they did not find evidence to reject the hypothesis of an unit root for many variables, except for the rate of unemployment. Since then, several investigations have used alternative methods to empirically assess the randomness of the trend and/or the permanent component.

It should be pointed out that the distinction between deterministic versus stochastic trends is not trivial for business cycles analysts. If a variable follows a random walk, the effect of temporary shock will not dissipate after several years, but instead, will be persistent. Hence, fluctuations will not longer be considered as transitory but, rather, as permanent. Time series variables exhibiting a stochastic trend will have no tendency to return to their previous level of long-run growth. The presence of a stochastic trend implies that fluctuations are the result of shocks not only to the transitory component (cyclical), but also to the trend component. This implies that a substantial fraction of the quarterly variation in real GNP is associated with movements in the long-run rather than purely transitory fluctuations.

Another implication of a stochastic trend or a variable following a random walk (usually with a drift), is that the regression of one against another can lead to spurious correlation. This means that the well-known Gauss-Markov theorem would not hold, since the level of a variable following a random walk does not have finite variance. Additionally,

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OLS parameters are not consistent estimators as assumed in the classical regression model. If a random walk is present, detrending the variable before running a regression would not help since the detrended series will be non-stationary. In this case, the first differencing procedures will only provide a stationary series.

Econometricians have been using several methods in evaluating the presence of a stochastic trend or random walk. Other methods have been used to decompose the fraction attributed to trend (permanent component) relative to the transitory. The methods mostly recognised are:

- (a) Unit Root Test
- (b) Measure of persistence
  - (i) Variance Ratio Test (VRT), and;
  - (ii) Infinite sum of moving average,  $A(1)$
- (c) Decomposing the components--permanent and the cyclical
  - (i) ARIMA procedures

This paper attempts to examine these methods for the annually and quarterly figures of GDP/GNP for Puerto Rico, with the aim of determining the presence of a stochastic trend or a random walk. The permanent and transitory component in real output figures will be also decomposed in the process.

The following is the plan for this paper. Section two will briefly review previous studies about this issue. Section three will discuss alternative methods of measuring persistence and of decomposing time series between permanent and transitory components. Section four will empirically estimate persistence and the relative important of each

component for the real output of Puerto Rico. Finally, section five will address the issue raised by Perron (1989) about the shifting or segmented trend for Puerto Rico's real output.

## **2. Review of Previous Studies**

This section attempts to summarise many of the empirical investigations done for United States, Canada, United Kingdom, among others.

### **2.1. Evidence for United States**

Nelson and Plosser (1982) using an ADF test, provide evidence that many U.S. annual macroeconomic time series are reasonably characterised as a random walk with a drift, or, by stochastic trends<sup>2</sup>. This finding contradicts that of the traditional view of business cycle- i.e; a time series variable, such as real output, fluctuating around a deterministic trend. Shocks, therefore, have enduring effects that do not die away immediately, but rather, they persist over time.

Campbell and Mankiw (1987a,1987b) corroborated Nelson and Plosser conclusions but using quarterly instead of annual data. The authors found that U.S. output is less persistent in the pre-war period than in the post-war period<sup>3</sup>. A similar conclusion was

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2. Nelson and Plosser failed to find any stochastic trend in the rate of unemployment. They did not find any evidence to reject the stochastic trend (unit root) in real and nominal output prices, wages, monetary variables and asset prices. The authors used annually data over a periods of 60 to 120 years. The starting date were 1860 and 1909 and ending is 1970.

3. According to the authors, fluctuations appear particularly transitory before 1929. For the period 1986-1929, the innovation attributed to the permanent component is 29%, while for the period 1930-1984 is 71% (window size k=10). For large window size, the persistent remains significant for the second period (see page 875).

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reached by Stock and Watson (1986), W. Schwert (1987), and De Long and Summer (1987)<sup>4</sup>. Perron and Phillips (1987) also tested Nelson and Plosser finding but only for real per capita GNP and came to a similar conclusion. However, when Friedman and Schwartz's annual estimated series were used (from 1869 to 1975), the authors found no evidence of random walk with a drift for the periods between 1869-1919 and 1869-1940. Finally, the authors corroborated different behavior before and after World War II. Malliaris and Urrutia (1990) using a variance ratio test, examined the data sample used by Nelson and Plosser and found that many U.S. macroeconomic variables have significant random walk components, with the exception of the unemployment rate, real wages, real per capita GNP and industrial production (window size  $k=30$ ). Nominal GNP, GNP deflator, consumer price index, wages, money stock and bond yield, exhibited significant permanent components.

In contrast, Watson (1986) and Clark (1987) analysed persistence in U.S. output using unobservable component models and discovered only a small permanent component. Clark, for instance, found that at least half of the quarterly innovation in United States variables can be attributed to the transitory component. He also concluded that less than 20% of the variance of the annual growth rate of the real per capita GNP between 1869-1986 was attributed to the permanent component.

On the other hand, a study by Pischke (1992) that compared the persistence via variance ratio annually in postwar U.S. GNP versus quarterly GNP found a lower permanent component in the former (window size,  $k=20$  years). A smaller permanent component was

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4. An important methodological objection was raised by Jaeger (1990) in relation to the study by Stock and Watson. According to this author, the results from Stock and Watson, including previous studies also, are misleading because linear trend interpolation was used for constructing the U.S. prewar output series.

found in per capita GNP than for real GNP. For annual per capita GNP permanent component is less than 5 percent while for quarterly per capita it was about 12% ( $k=20$  years). However, for annual real GNP the permanent component was 41 percent and the quarterly real GNP was 64 percent. According to this author the result is one of the explanatory factor in the apparently contradictory conclusions presented in studies by Campbell and Mankiw, and Cochrane<sup>5</sup>.

## **2.2 Evidence for United Kingdom and Canada**

Several papers also examined the random walk hypotheses on real output of U.K. and Canada. For the United Kingdom, Walton (1988), Mills and Taylor (1989), Mills (1991) have concluded that real output has a unit root, with innovations to output being highly persistent. Mills (1991), going one step further indicates that output innovations have a tendency to be magnified over time, rather than die away quickly. This result contradicts previous cross- country results by Campbell and Mankiw (1989), that showed a low persistence in U.K. output.

For Canada, Serletis examined on an annual basis the properties of Canadian real output from 1870 to 1985. Using the Dickey-Fuller test, Serletis found that shocks to the Canadian economy have a relatively large permanent effect of about 60 percent.

Lee and Siklos (1991) examined the sensitivity of the unit root for post-war quarterly Canadian macroeconomic series to seasonally adjustment procedures (i.e., ARIMA X-11)

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5. The author showed that the hypothesis of a trend-break in U.S. GNP and the population data explain also the differences in the variance ratio between real GNP and per capita GNP. The second explanatory element is the effect of time aggregation. Using a Monte Carlo experiment he indicated that quarterly data generated a higher degree of persistence than the annual figures.

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applied by government agencies in Canada. They found that the use of seasonally unadjusted data, with one possible exception, does not influence the unit root test results in a seasonally adjusted data. Therefore, seasonal factors did not affect the conclusion of a unit root for Canadian output.

### **2.3 International comparison**

Several across-country studies also compared the existence of a stochastic trend among different countries (Clark(1989), Campbell and Mankiw (1989), Kordemi and Meguire (1990), Cogley (1990), Deibold and Rudebush (1989) Cheung and Lai (1992), *inter alia*).

Campbell and Mankiw (1989), showed evidence of high persistence in quarterly output fluctuations for U.S. and other G-7 countries (Canada, France, Germany, Italy, United Kingdom). United Kingdom showed a lower persistence component than other countries. For six out of seven countries, the point estimates indicate that a 1 percent shock to output should change the long-run univariate forecast of output by over one percent. Output for Canada, France, Germany, Italy and Japan had a large persistent component. However, the authors were careful not to rule out the possibility of a very slow return to the trend. On the other hand, Kormendi and Meguire (1990) confirmed the conclusion that the long span of data for U.S., when viewed in isolation, provide questionable evidence in support of an unit root (random walk). When viewed as a whole, among a variety of countries, U.S. output is consistent with the presence of a unit root.

Diebold and Rudebusch and Cheng and Lai extended the impulse response analysis to fractionally integrated processes. The first two found evidence of fractional integration and

reported that the effect of shocks to output will quickly disappear (persistence is low). The later two found that U.S. and U.K. exhibited much less persistence in output fluctuations than the other industrialised G-7 countries. Clark (1989) reports similar output behaviour for several other major developed countries. Cogley (1990) estimated variance ratios for the rates of growth of OECD countries over the period 1871-1985 and found evidence supporting a relative stable long-run growth in United States than in other countries. The variance ratio for the U.S. was half of that for countries with a high persistence component such as Australia, France and Italy. Nonetheless, U.S. estimates were relatively small compared to the estimates of Cochrane.

Haan and Zelhorst (1993) performed unit root tests for annual per capita output of twelve OECD countries for the period of 1870 to 1989 and found that this test cannot be rejected for all of them, except for the United States. However, when it allows for a trend shift or time-break, the unit root tests failed in most countries of the sample.

In summary, despite a great deal of efforts devoted to the examination of the issue of persistence or stochastic trend, the evidence seems to be inconclusive. The use of a variety of models, the frequency of the data (quarterly versus annual), the problem of pre-war data estimates, among others, require further studies in the field of unit root and trends.

### **3. Brief summary of the methods**

The following section will summarise the alternative methods commonly used to evaluate the presence of a stochastic trend. This section attempts to apply these methods to evaluate the presence of a stochastic trend or random walk in real GNP or real GDP of Puerto

Rico (hereafter, PRGNP and PRGDP, respectively). Table 1 depicts some general features of PRGNP/PRGDP figures of Puerto Rico.

### 3.1 The Unit Root Test

One commonly recognised way to determine the presence of a stochastic trend is by the unit root test. In this test a particular time series has an unit root (stochastic trend) if the null hypothesis cannot be rejected. The alternative hypothesis is that the series follows a trend stationary process. Two of the most commonly used tests are Dickey-Fuller (DF) or Augmented Dickey-Fuller (ADF), and the Phillips-Perron test. The DF or ADF will be the only one applied to the real output of Puerto Rico.

Table 1  
Summary of Real GNP/GDP Series

Variable	Frequency	Time Span	N
PRGNP	Annual	1950 to 1993	44
PRGDP	Annual	1950 to 1993	44
PRGNP	Quarter *	Q3:61 to Q4:90	116
PRGDP	Quarter *	Q3:61 to Q4:90	116

\* Estimated by the author by interpolation. See paper by Litterman (1988).

The DF or ADF test is based on the t value of the coefficient of  $y_{t-1}$  in a regression:

$$\Delta y_t = \mu + \alpha y_{t-1} + e_t \quad (1)$$

This model is called DF approach regression because contains no lags. ADF model includes m lags:

$$\Delta Y_t = \mu + \alpha Y_{t-1} + \sum_{i=1}^m \beta_i \Delta Y_{t-i} + e_t \quad (2)$$

where m is chosen to be sufficiently large for the residual  $e(t-1)$  to be reduced to white noise<sup>6</sup>.

### 3.2 Measure of persistence

Another method of evaluating the presence of a stochastic trend (random walk) is by determining the degree of persistence of the variable. According to Campbell and Mankiw (1989), persistence can be defined as a shock to an economy in which the effects last for more than one period. This may imply that a significant portion of output fluctuations is due to the permanent (trend) component rather than a transitory (temporary).

Cochrane (1988) have suggested an alternative measure by using the variance ratio ( $VR_k$ ) and its testing procedure known as VR test (VRT). This  $VR_k$  is estimated non-parametrically and is robust to heteroscedasticity and non-normal random disturbances.

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6. A formal test to select the value of m is by Akaike's Information Criterion (AIC) and/or Schwarz-Bayes Information Criteria (SBIC).

The  $VR_k$  is given by the following equation:

$$VR_k = \frac{1}{k+1} \cdot \frac{\text{Var} (Y_t - Y_{t-k})}{\text{Var} (Y_t - Y_{t-1})} \quad (3)$$

in which the numerator,  $\text{Var} (y_t - y_{t-k})$ , is the variance of the  $k$  differences, and the denominator is the variance of the first differences of the selected series.  $k$  is the number of differences considered or the window size, measured in terms of the number of period. Persistence is measured by the limit of the  $VR_k$ . For example, for a random walk,  $VR_k$  is one while for any trend stationary  $VR_k$  is zero. The term  $(1/k+1)$  times  $\text{Var} (y_t - y_{t-1})$  is settled down to the variance of the shock to the random walk component, that is, the permanent component.

On the other hand, following common practice in the literature, the measures are corrected for small sample bias by multiplying the  $VR_k$  by an adjustment factor  $(N-1)/(N-k-1)$ , where  $N$  is the sample size and  $k$  is the window size <sup>7</sup>.

The asymptotic standard error (ASE) for the VR estimate is given by:

$$ASE = \sqrt{\left(\frac{4}{3}\right)\left(\frac{k}{n}\right)} \cdot VR_k \quad (4)$$

The  $VR_k$  is related also with  $A(1)$ , a measure of persistence obtained from the spectral density at frequency zero;

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7. For further details see Albert Jeager (1990), and Campbell and Mankiw (1987a,1989).

$$A(1) = \sqrt{\frac{V}{1-R^2}} \quad (5)$$

where V is the limiting VR<sub>k</sub> (when k approaches to infinity)

$$V = \lim_{k \rightarrow \infty} VR_k \quad (6)$$

and [ $R^2 = 1 - \text{var}(e(t)) / \text{var}(dy(t))$ ] that can be replaced by the square of the first autocorrelation coefficient. Therefore, the A(1) equation becomes:

$$A(1) = \sqrt{\frac{VR_k}{1-\sigma^2}} \quad (7)$$

According to Campbell and Mankiw,  $\sigma^2$  is an underestimate of  $R^2$ , except for an AR(1). Thus, this estimate tends to understate the value of A(1). If A(1)=1, then, there is a random walk process, but if A(1)=0, there is a stationary or deterministic trend process.

### 3.3 Infinite sum of moving average

A useful way of analysing an ARMA (ARIMA) model is by the moving average representation which is equivalent to tracing out the dynamic effects of a variable over time given a particular shock. Formally, suppose that the changes in log of real GNP (approximately, the rate of growth) is a stationary ARMA process with a moving average representation:

$$\Phi(L) = 1 - \Phi_1 L - \Phi_2 L^2 - \dots - \tag{9}$$

where,

and,

The process can be rearranged to arrive at the moving average representation or

$$\Theta(L) = 1 + \Theta_1 L + \Theta_2 L^2 + \dots + \tag{10}$$

impulse response function (IRF) for  $\Delta Y_t$ :

$$\Delta Y_t = \Phi(L)^{-1} \Theta(L) e_t = \tag{11}$$

$$A(L) e_t = (1 + A_1 L + A_2 L^2 + \dots) e_t \tag{12}$$

The difference stationary approach has different implications regarding the effects of a given shock on the level of real GNP. If the change in real GNP is stationary, then the

$$\Phi(L) \Delta Y_t = \Theta(L) e_t \tag{8}$$

summation of  $A_i^2$  is finite, implying that the limit of  $A_i$  as  $i$  goes to infinity is zero. If one derives the process, for the level of real GNP we obtain the moving average representation:

$$Y_t = (1 - L)^{-1} A(L) e_t = B(L) e_t \tag{13}$$

where,

$$B_i = \sum_{j=0}^i A_j \tag{14}$$

The limit of  $B_i$  is the infinite sum of  $A_j$  values, which can also be written as  $A(1)$ . The ultimate impact of the shock on the level of GNP equals the infinite sum of the moving average coefficients. If  $A(1)=1$ , the process follows a random walk, that is, the effect of any shock is exactly permanent. For any variable which is stationary around a deterministic trend,  $A(1)$  equals zero.

### **3.4 Decomposing the components**

#### **3.4.1 ARIMA models**

The empirical approach in decomposing the permanent and stationary components using ARIMA (or ARMA), relies basically on the assumption of the degree of correlation between both components. Since correlation between both components cannot be estimated directly from any single series, business cycles analysts have considered two extreme assumptions: (1) both components are perfectly correlated, i.e; Beveridge and Nelson (1981), or (2) both components are uncorrelated, i.e; Watson (1986). If components are perfectly correlated, the innovations on GNP arises from the same source; if components are not correlated, innovations on GNP come along for each component and they will be modelled separately.

Beveridge and Nelson (1981) have developed an appropriate method in decomposing a time series into its permanent and cyclical components using ARIMA procedures, but under the assumption of perfect correlation. In general, the importance of ARIMA model in such decomposition relies that a given structure in ARIMA (p,1,q) representation, contains a random walk with drift plus a stationary component. Using ARIMA procedures one can reduce all unforeseen economic events into a single innovation. In the Beveridge and Nelson approach, both the permanent and transitory components are based on these innovations.

The other approach (zero-correlation) was presented by Watson using ARMA or ARIMA (p,0,q) procedure. Since neither component is observed directly, this model can be called an unobserved component ARIMA (UC-ARIMA). Watson fits this type of model, where the stationary component is assumed to be a second order autoregression, i.e; to be ARMA (2,0). His model is as follows:

$$\begin{aligned}
 y_{(t)} &= y_{p(t)} + y_{s(t)} \\
 y_{p(t)} &= a + y_{p(t-1)} + e_{p(t)} \\
 y_{s(t)} &= b \cdot y_{s(t-1)} + c \cdot y_{s(t-2)} + e_{s(t)} \\
 \text{Cov}(e_{s(t)}, e_{p(t)}) &= 0
 \end{aligned} \tag{15}$$

The permanent component,  $y_{p(t)}$ , is written as a random walk with a drift while the stationary,  $y_{s(t)}$ , is predicted using two lags.

### 3.4.2 ARIMA and the Covariance Matrix

Another more simple computational method was suggested T. Hall, W. Fields and M. Fields (1989), thereafter HFF. In HFF a simple procedure is proposed in which the

variance of output growth is allocated between the trend and the cycle components. Both components are computed by the ratio of the covariance between the growth rate of output and the growth rate of each of the components to the total variance of the growth rate of output.

The procedure suggested by HFF can be summarised as follows. Let us say  $Y$ , the natural logarithm of the real GNP or real GDP, is decomposed into the trend component (T),

$$Y = T + C \quad (16)$$

and the cyclical component (C).

Then, the first differences for each component as well as output will closely represent the rate of growth of the real GNP and its components. Using the expression for the variance of the sum, we have the following expression:

$$\cdot VAR(dY) = VAR(dT) + VAR(dC) \quad (17)$$

after some mathematical operations, the authors ended with the following expressions:

$$\tau_t = \frac{Cov(dY, dT)}{Var(dY)}; \quad (19)$$

$$\tau_c = \frac{Cov(dY, dC)}{Var(dY)} \quad (20)$$

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where  $t_t$  is the proportion of the variance in the output (growth rate) attributable to the change in the trend component and,  $t_c$  the counterpart and for the cyclical component. Therefore,  $t_c + t_t$  must be equal to one. HFF using this approach found a similar proportion of each components as in Beveridge and Nelson's study. The proportion attributed to changes in the trend component was 76.4% and to the cyclical component, 23.6%.

#### **4. Measuring Persistence and Components of the Real Output of Puerto Rico.**

This section attempts to estimate the degree of persistence or test for the presence of a stochastic trend in the real output series of Puerto Rico. There are several ways to measure real output in Puerto Rico. Table 1 summarised the real output figures that will be assessed in the is section. The annual data of real GDP/GNP are official figures from the National Account System of the Planning Board, while the quarterly were derived from an interpolation based upon the monthly index of economic activity<sup>8</sup>. Unfortunately, there are no official quarterly figures on real GDP/GNP or any other national account variable of Puerto Rico. Researchers sometimes use interpolation from United States GDP or from any available monthly (or quarterly) variable from Puerto Rico. Since many economic series exhibit a trend coupled with a larger dispersion when their absolute values increase over time, many researchers transform the series into natural logarithm.

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8 For further details please see Chapter 5, Section 5.2.

#### **4.1 Unit Root Test**

Tables 2 and 3 depict the DF and ADF test for the presence of a unit root in the real GDP and GDP of Puerto Rico. The tables also show the values of AIC and SBIC, which provide useful criteria necessary to select the number of lags in the regression that exhibited white noise residuals. AIC and SBIC are minimised at one lag for the real GDP and for the real GNP, SBIC is minimised at two lags but for AIC real GDP at four lags. Therefore, the unit root test provides no statistical evidence to reject the presence of a random walk or a stochastic trend in the annual real GNP and GDP figures of Puerto Rico.

Considering the quarterly figures, the ADF test does not provide statistical evidence to reject the presence of an stochastic trend. Using a critical value of 95 %, actual values were not found significant in any lags. The AIC and SBIC values are minimised for both output figures at four lags. Therefore, ADF test supports statistical evidence supporting the presence of a random walk or a stochastic trend in the quarterly real output of Puerto Rico.

Table 2  
 Unit Root Test  
 Dickey -Fuller Test (with trend)  
 Real GNP and Real GDP for Puerto Rico  
 From FY 1950 to FY 1993

Unit Root Test	Real log GNP			Real log GDP		
	$t_a$ value	AIC	SBIC	$t_a$ value	AIC	SBIC
ADF(lags)						
DF	-0.200	-4.391	-7.106	-0.418	-3.998	-6.713
DF(1)	-1.248	-4.748	-7.420	-1.072	-4.098*	-6.770*
ADF(2)	-0.499	-4.843	-7.458*	-1.105	-3.988	-6.563
ADF(3)	-0.927	-4.842	-7.427	-0.952	-3.956	-6.491
ADF(4)	-0.756	-4.893*	-7.432	-0.765	-3.892	-6.385
ADF(5)	-0.940	-4.830	-7.323	-0.915	-3.858	-6.304

\* *minimised value.* 95 % critical value is -3.52 for  $t_a$  value

Table 3  
 Unit Root Test  
 Dickey -Fuller Test (with trend)  
 Real GNP and Real GDP for Puerto Rico  
 From Quarterly 1961:01 to 1990:03

Unit Root Test	Real log GNP			Real log GDP		
	$t_a$ value	AIC	SBIC	$t_a$ value	AIC	SBIC
ADF(lags)						
DF	-2.980	-6.330	-9.099	-3.196	-5.858	-8.623
ADF(1)	-2.453	-6.427	-9.167	-2.731	-5.965	-8.707
ADF(2)	-1.862	-6.451	-9.168	-2.052	-5.995	-8.712
ADF(3)	-1.685	-6.473	-9.165	-1.906	-5.991	-8.683
ADF(4)	-2.023	-6.576*	-9.183*	-2.214	-6.062*	-8.723*
ADF(5)	-2.090	-6.493	-9.134	-2.198	-6.035	-8.676
ADF(6)	-2.505	-6.554	-9.170	-2.605	-6.108	-8.723
ADF(7)	-2.089	-6.531	-9.120	-2.313	-6.058	-8.675

\* *minimised value.* 95 % critical value is -3.45 for  $t_a$  value

#### **4.2 Measures of Persistence**

Table 4 shows the Variance Ratio (VR) test for real output in Puerto Rico. It also shown the VR adjusted for sample size and the standard error of the estimates. For the annual real GNP/GDP figures, adjusted VR depicts not only a significantly high value for window sizes (years) of 10, 15, and, 20, but also extraordinary large values for the standard errors. This implies that the random walk component cannot be measured accurately at these window size. However, analysing VR and the corresponding standard errors for window sizes of 30 and 40 years, it can be concluded that the permanent components in annual real GNP and GDP of Puerto Rico support persistence. In other words, these variables follow a random walk process.

A similar conclusion was also derived using quarterly estimates of real GNP and GDP. The VR for quarterly GNP and GDP showed large values for window sizes of 15, 20, 30 and 40 quarters, and in addition depicted large values for their corresponding standard errors. Nevertheless, when standard errors are relative low--i.e., window size of 10 and 15, VR supported that quarterly estimates of real GNP and GDP both follow a random walk process.

Table 4  
Variance Ratio Test  
Real GDP and GNP of Puerto Rico

Variable/ Window Size=k	k=1	k=5	k=10	k=15	k=20	k=30	k=40
Annual PRGNP	1.0000 (0.1946)	1.831 (0.6453)	3.2945 (1.409)	5.8301 (2.581)	4.750 (1.989)	0.9479 (0.2744)	1.665 (0.1275)
Annual PRGDP	1.0000 (0.1946)	2.534 (0.8932)	5.053 (2.1612)	5.507 (2.437)	7.5412 (3.160)	2.112 (0.6113)	2.2562 (0.1741)
Quarter PRGNP	1.0000 (0.1072)	2.372 (0.5685)	3.455 (1.1714)	4.2084 (1.747)	4.5717 (2.192)	5.198 (3.052)	5.291 (3.588)
Quarter PRGDP	1.0000 (0.1072)	1.6443 (0.394)	2.218 (0.7521)	2.552 (1.060)	2.514 (1.205)	2.441 (1.433)	2.572 (1.744)

Standard Error in parenthesis. Annual figures are adjusted for the sample side bias. See text for formula.

Table 5 shows the measure of persistence developed by Cochrane. Once again, regarding all window size, both real GDP and GNP exhibited large values of  $A(1)$ . This measure of persistence  $A(1)$  also supports a random walk process (stochastic trend) for both real output figures (annually or quarterly).

### 4.3 Infinite sum of a moving average

Persistence can also be approached by analysing the coefficients of an infinite sum of a moving average. This section is based upon Campbell and Mankiw's Box-Jenkins ARIMA (2,1,2) for the log of real GNP/GDP of Puerto Rico. Table 6 shows the parameters and the diagnostic statistics, Q test (Portmanteau Test) for ARIMA (2,1,2) model. The last column from left to right, refers to the "long-run response" (LRR) which refers to the sum of moving average coefficients or the ultimate impact of the shock on the level of the real

GNP/GDP. As can be seen, annual real GNP/GDP of Puerto Rico exhibited a A(1) limit values of 0.830 and 5.92, respectively. These values also support a random walk process although they are lower than Cochrane's values of persistence.

LRR for quarterly GNP and GDP support also the previous conclusions. Values of quarterly GNP was 5.89, and for the real GDP was 1.62. The value for the real GNP was significantly larger than that of Cochrane's at any given window size, while was quite similar to that of real GDP. Nonetheless, values from LRR confirmed a persistence process at an infinitive horizon.

Table 5  
Cochrane's Measure of Persistence, A(1).

Variable/ Window	k=1	k=5	k=10	k=15	k=20	k=30	k=40
Annual PRGNP	1.367	1.829	2.453	3.263	2.944	1.316	1.739
Annual PRGDP	1.105	1.739	2.455	2.563	3.000	1.587	1.642
Quarter PRGNP	1.099	1.693	2.043	2.255	2.350	2.506	2.528
Quarter PRGDP	1.098	1.408	1.636	1.755	1.742	1.716	1.762

Table 6  
Box-Jenkins ARIMA (2,1,2) Parameters and Diagnostic Statistics

Variable	AR(1)	AR(2)	MA(1)	MA(2)	Q <sup>20</sup>	LRR*
Annual PRGNP	0.3730	0.0745	-0.6050	0.06360	11.205 (0.738)	0.83
Annual PRGDP	0.8713	-0.7370	0.5860	-0.8878	13.23 (0.584)	0.803
Quarter PRGNP	0.0336	0.7529	-0.3753	0.6340	23.93 (0.0663)	5.88
Quarter PRGDP	-0.131	0.6087	-0.5760	0.4223	28.18 (0.0205)	1.62

Q<sup>20</sup> refers to Chi-square test of first 20 residuals autocorrelation (probability of a large value given white noise in parenthesis). LRR is A(1) or long-run response or infinitive sum of moving average.

The long-run response process can be analysed by the univariate impulse response functions (IRF), which portray cumulative values characterising by a permanent shock (see Graph 1 and Graph 2). Annual real GNP approaches to the long-run value in a shorter time than the annual real GDP.

#### 4.4 Decomposition by components.

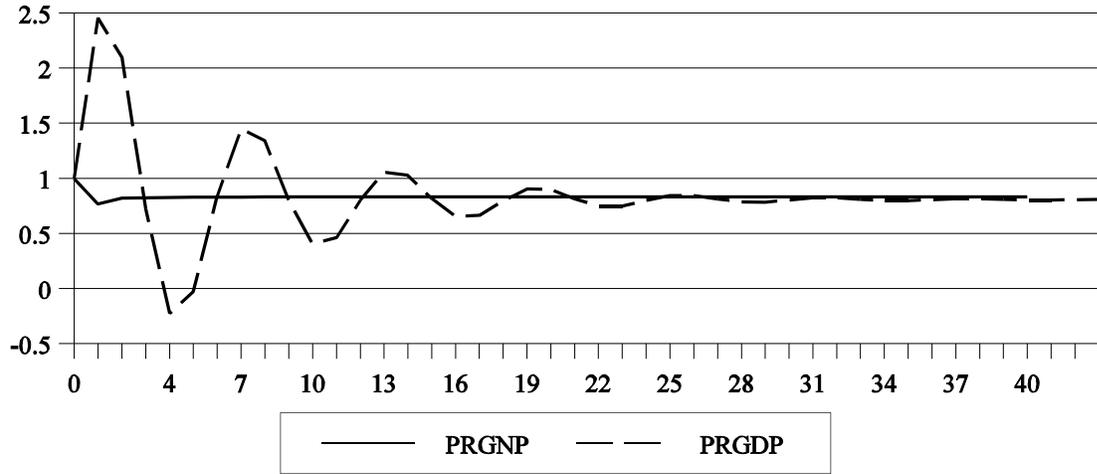
##### 4.4.1 ARIMA models

Although no single method exists to determine how much of the real GNP or GDP variations in Puerto Rico is due to the permanent and transitory components, there are some empirical approaches to address this issue.

**Graph 1**

**Long-Run Response of Real Output in Puerto Rico- Cumulative**

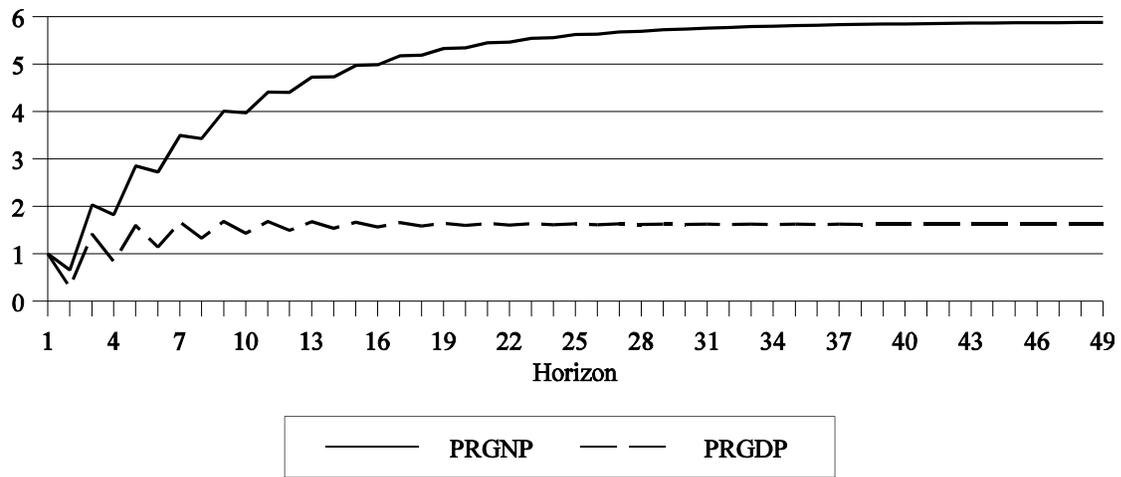
**Annual data**



**Graph 2**

**Long-Run Response of Real Output in Puerto Rico- Cumulative**

**Quarterly data**



Beveridge and Nelson, for instance, suggested an ARIMA (0,1,1) procedure. Fitting such model to the quarterly real GNP/GDP figures, the following results were found;

For real GNP:

$$dy_{(t)} = 0.00970 + e_{(t)} - 0.4165 e_{(t-1)} \quad \text{S.E.} = .0103377$$

$$Q_{20} (114) = 48.44 (0.00013)$$

for real GDP;

$$dy_{(t)} = 0.01242 + e_{(t)} - 0.40685 e_{(t-1)} \quad \text{S.E.} = .0127662$$

$$Q_{20} (114) = 39.0 (0.00285)$$

Using Beveridge and Nelson's decomposition, the permanent and the transitory components are shown in Table 7. As can be seen, permanent components in both real output figures are about 14 percent of quarterly real output growth. This result confirms the modern position that short-term movements on real output are attributed to a large amount of quarterly output movements which shift the aggregate supply or potential GDP<sup>9</sup>.

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9. In fact, Boschen and Mills (1990) summarised a group of studies that confirm shifts in potential GNP which explain between 27 percent to 72 percent of observed GNP variations in U.S. These results are presented later.

Table 7  
Permanent and Transitory Components  
Beveridge and Nelson Approach  
ARIMA (0,1,1)

Variables *	COMPONENTS	
	Permanent (%)	Transitory (%)
Real GNP	16.3	83.7
Real GDP	14.6	85.4

\* Quarterly figures

For the "zero-correlation" case, the transitory component was modeled as a second order autoregression ARMA (2,0), and the permanent component as a random walk with a drift, ARMA (1,0).

Real GDP and GNP are explicitly represented by the sum of these two independently estimated components. Results for quarterly real GNP and real GDP figures are presented below. For the log of real GNP the permanent component is:

$$y_{p(t)} = -0.06719 + 1.00911 y_{p(t-1)}$$

$$S.E.(e_{p(t)}) = 0.01317 \quad Q^{20} (114) = 184.355 (8.8819 E-15)$$

the transitory component is:

$$y_{s(t)} = 1.60651 y_{s(t-1)} - 0.60315 y_{s(t-2)}$$

$$S.E.(e_{s(t)}) = 0.010643 \quad Q^{20} (113) = 57.75 (2.4534E-6)$$

and the covariance of the residuals is

$$COV ( e_{p(t)}, e_{s(t)} ) = 0.00011.$$

for the log of real GDP, the permanent component is:

$$y_{p(t)} = -0.07571 + 1.01028 y_{p(t-1)}$$

$$\text{S.E.} (e_{p(t)}) = 0.015996 \quad Q^{20} (114) = 109.3 (4.2188\text{E-}15)$$

the transitory component is:

$$y_{s(t)} = 1.57164 y_{s(t-1)} - 0.56746 y_{s(t-2)}$$

$$\text{S.E.} (e_{p(t)}) = 0.0133389 \quad Q^{20}(113) = 52.04 (2.03907 \text{ E-}5)$$

and the covariance of the residuals is:

$$\text{COV} (e_{p(t)}, e_{s(t)}) = 0.00017$$

According to Stock and Watson (1988), the fraction of the permanent component attributed to variations in the quarterly GDP or GNP can be estimated by a "R-square" statistics. This statistic roughly measures the fraction of GNP/GDP variations attributed to shifts in the trend. Yet an observational equivalent of such zero-correlation UC-ARIMA model was also developed by Stock and Watson and presented in their research (footnote 8, p. 157). Based on this equivalent method, the permanent component for the quarterly real GNP is 35.5 percent, and for the real GDP is 34.9 percent. However, this model specification is not appropriate for Puerto Rico real GDP (GNP) because residuals (innovations) are correlated violating the main assumption of UC-ARIMA. Both correlation coefficients are over 75 percent.

#### **4.4.2 ARIMA and the covariance matrix.**

Another alternative method of estimating the portion of the variation in output attributed to the permanent component (trend) was presented by HWW who estimated an ARIMA (4,1,0) model for the real GNP of United States and found similar proportions as Beveridge and Nelson.

Table 8  
Permanent and Transitory Components  
ARIMA and covariance matrix method

Variable (Quarterly)	ARIMA (2,1,0)		ARIMA (4,1,0)		ARIMA (6,1,0)	
	P	T	P	T	P	T
PRGNP	48.2 %	51.8 %	55.2 %	44.8 %	59.2 %	40.8 %
PRGDP	49.0 %	51.0 %	56.2 %	43.8 %	59.5 %	40.5 %

Note: P= Permanent component; T= Transitory Component. Both components are calculated using the procedure of HWW (1989).

In this section, three alternative ARIMA specifications will be considered: ARIMA (2,1,0), ARIMA (4,1,0) and ARIMA (6,1,0). The results for quarterly GNP and GDP are shown in Table 8. For the three ARIMA model specifications, the variation in output attributed to its permanent component is significant. For the ARIMA (4,1,0), for example, the trend component of the quarterly real GNP is 55.2% and for the PRGDP is 56.2%. Meanwhile, the portion attributed to the variation of the real GNP and GDP in the ARIMA (6,1,0), is 59.2% and 59.5%, respectively. Despite the portion attributed to each component seeming to be a function of the ARIMA model specification, the trend component in each case is clearly significant. Once again, the estimates of the proportions of the permanent component that causes real output variations tend to support the RBC hypothesis rather than the conventional view of business cycles.

A comparison of the estimates of Puerto Rico with those of the United States is shown in Table 9. Estimates of Puerto Rico permanent component are quite similar to that of King, Plosser, Stock and Watson, but somewhat lower than Shapiro and Watson.

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However, they are also quite larger than that of Blanchard and Quah, Boschen and Mills, and the traditional theory.

## **5. Conclusions on the stochastic trend and persistence**

In summary, the alternative methods suggested to test the presence of a random walk or a stochastic trend, were applied to real GDP or GNP figures of Puerto Rico. Meanwhile, alternative models were utilised in order to estimate the fraction of real output attributed to the permanent (trend) or to the transitory component (cyclical). All methods tend to support the RBC proposition of the presence of a stochastic trend or to a relatively large fraction of quarterly real output variations explained by the permanent component.

Table 9

Percent of Variance in Quarterly real GNP  
Due to Variability in the Permanent Component

METHODS	PERCENT
TRADITIONAL	1
ISOLATING TREND	
Blanchard and Quah	36
Judd and Trehan	36
King, Plosser, Stock and Watson	54
Shapiro and Watson	72
EXAMINING EXOGENOUS VARIABLES	
Boschen and Mills	27
Simple Growth	38
VARIOUS FOR PUERTO RICO	
Beveridge and Nelson	15
AC-ARIMA	36
ARIMA (2,1,0)	48
ARIMA (4,1,0)	55
ARIMA (6,1,0)	59

Source: Boschen J. and L. Mills (1990). Estimates of Puerto Rico taken from this study.

## 6. Shifting and Segmented Trend Analysis

### 6.1 The model

A several authors have questioned the strength of the evidence in favor of stochastic trends [Perron (1989) Rappoport and Reichlin (1989), Guilkey and Schmidt (1989),inter alia]. These authors have re-examined many series characterised by a unit root and found

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surprising evidence when exogenous shocks such as 1930's Great Depression or oil price hikes on 1973, are allowed into the testing procedures. The evidence in favor of a stochastic trend, they claimed, was overstated. In particular, series such as real GNP could be more accurately described by deterministic trends that occasionally shift or change their slopes, that is, as a deterministic broken line trend. Balke and Fomby (1990) argued that permanent shocks are better considered as infrequent shocks with random or unpredictable occurrence. For instance, the 1973 oil price shocks may have had some important effect on output; yet these events do not occur in periodical patterns or every period.

Perron, examining the U.S. macroeconomic series used by Nelson and Plosser, argued that most time series are trend stationary allowing for one- time break in the slope or in the intercept (or both) of the trend function. In fact, Perron found evidence that does not support a unit root when the 1929 Great Crash and as well as the 1973 oil price hikes are incorporated into the trend function. He showed that the null hypothesis of a unit root was rejected at a high level of confidence for 11 out of 14 series if one allows for three time breaks in the function. For example, for the quarterly real GNP series a time-break occurs in 1973 due to oil price shock and takes the form of a change in the slope. The Great Depression, takes the form of a change in the intercept. Hence, the analysis of the time-break function led the author to reject the null hypothesis or the presence of an unit root. Obviously, this result challenged that of Nelson and Plosser and many other business cycles analysts who support the hypothesis of a unit root or stochastic trend time series.

On this basis, an extra procedure for the test for structural changes and/or segmented trends for the real output of Puerto Rico was included in the following section.

## 6.2 The empirical evidence to Puerto Rico

This section will examine Perron's conclusion for real output in Puerto Rico. As Perron did, it is postulated that certain shocks such as the 1973-74 oil price hikes, can be considered as exogenous, and thereby, removed from the noise function. The unit root will be tested but with some particular appropriate modifications in the DF or ADF regression.

The following OLSQ regressing equation was considered:

$$y_t = \mu + \phi DU_t + \beta trend + \gamma DT_t + \delta \quad (19)$$

where,  $DU_t = 1$ , and  $DT_t = t$  if  $t > T_B$  and 0 otherwise, and  $D(t_b)_t = 1$  if  $t = T_B + 1$  and 0 otherwise. For the annual real GNP/GDP figures, the fiscal year (FY) 1974 was taken as a time break because it attempts to capture the shock of 1973-74 oil price hike. Given that this oil-price hike started at the last quarter of 1973 (October, 1973), it is reasonable to assert that its main effects were felt immediately after, say FY 1974. As a result the two regression models were tested, one for annual data and the other for the quarterly data.

For the quarterly real GNP and GDP, the hypothesis of unit root and time break was also examined. The selected time-break is the first quarter of 1974.

In testing the null hypothesis of a unit root, it is now necessary to compute the value of  $\lambda$ , according to Perron procedure.  $\lambda$  is the ratio of pre-break sample size ( $T_B$ ) to total sample size ( $T_B/T$ ). Therefore, the lambda parameter is between 0 and 1. Perron developed a table of critical values which allows to test the unit root hypothesis when exogenous shocks are taken into account. Given the break ratio, to be exogenous, the  $\alpha = 1$  or equivalent,  $t_\alpha$  value, will be compared with the critical value tabulated by Perron over

different values of  $\lambda$ .<sup>10</sup> The  $t_\alpha$  statistic on the parameters for the following respective hypotheses are also presented:  $\mu=0$ ,  $\Theta=0$ ,  $\delta=0$ ,  $\beta=0$ , and  $\alpha=1$ . In order to evaluate the significance of  $t_\alpha$  (1), the critical values are used with a value of  $\lambda$  of 60 percent for the annual output figures (time break set at 1974), and 40 percent for the quarterly figures<sup>11</sup>. In order to select the lags for each OLSQ regression, the Schwarz-Bayes Information Criterion (SBCI) was adopted. This statistic deals with the minimisation of the value of the determinant of the covariance matrix of the residuals.

The results for the unit root test, modified by Perron's procedures, are shown in Tables 10 and 11. The former table depicts the results for annual figures and clearly shows that the unit root hypothesis cannot be rejected neither for GNP nor GDP, at any of the three critical values 1%, 5%, and 10%. On the other hand, Table 11 shows the results for quarterly data. Once again, the test does not provide statistical evidence to reject the null hypothesis of an unit root, neither for GNP nor GDP. This is equivalent to saying that real output does not exhibit any mean reversion process, but rather, the shocks have strong persistence effects over the long-run horizon of real output in Puerto Rico, even when exogenous shocks such as oil-price hikes are taking into account.

These results shed some light on the presence of small or infrequent shocks. Not only have oil prices had long-term effects on the variability of the real level of output in Puerto

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10. To evaluate the significance of the t-value given a  $\lambda$ , we used the critical value presented by Perron in the Table VI.B.(p.1377) in Perron (1989), "The Great Crash, the oil price shock, and the unit root hypothesis" *Econometrica*, 57, 1361-1401.

11. For the annual data the observed  $\lambda$  value is  $24/44 = 56.82$  percent. For quarterly data this value is  $51/116 = 43.966$  percent. Given that critical values for  $\lambda$  are discrete, those values closest in the table are used.

Rico but also, other small and infrequent shocks have altered the long-run path of the real output in Puerto Rico.

Table 10

Test for Unit Root using Perron's Shifting Trend Procedure  
Annual real Puerto Rico GNP/GDP  
(FY 1950 to FY 1993; Time break =FY 1974)

OLS regression lambda=60%	Constant	Trend	Du <sub>t</sub>	DT	DT <sub>B</sub>	t <sub>α</sub> value	S.E.
Annual GNP a/	1.9780 (3.71)	0.0188 (3.85)	-0.0162 (-1.23)	-0.0124 (-3.82)	-0.0276 (-1.594)	-3.656	0.1534
Annual GDP b/	2.27 (3.52)	.0240 (3.56)	-0.030 (-1.75)	-0.012 (-3.40)	-0.044 (-1.63)	-3.449	0.0244

a/ SBIC minimised at 2 lags. b/ SBIC minimised at 1 lag. The t<sub>α</sub>-values at 1%\*, 5%\*\* , and 10%\*\*\*, given λ equals 60% are -4.88, -4.24 and -3.95, respectively. See Perron (1989), Op. Cit; Table VI.B, page 1377.

Table 11

Test for Unit Root using Perron's Shifting Trend Procedure  
Quarterly real Puerto Rico GNP/GDP  
(1961:03 to 1990:02; Time break = 1974:01)

OLS regression lambda= 40%	Constant	Trend	Du <sub>t</sub>	DT	DT <sub>B</sub>	t <sub>α</sub> value	S.E.
Quarter GNP a/	0.7764 (3.21)	0.0162 (3.03)	-0.0107 (-2.68)	-0.0103 (-2.76)	-0.0055 (-0.559)	-3.170	0.0000
Quarter GDP b/	0.690 (2.61)	0.1465 (2.292)	-0.0117 (-2.28)	-0.00061 (-1.72)	0.00396 (-0.319)	-2.551	0.011185

a/ SBIC minimised at 3 lags. b/ SBIC minimised at 4 lags. The t<sub>α</sub>-values at 1%\*, 5%\*\* , and 10%\*\*\*, given λ equals 40% are -4.81, -4.22 and -3.95, respectively. See Perron (1989), **Op. Cit**; Table VI.B, page 1377.

## **7. Conclusions**

This chapter sought to evaluate the presence of the stochastic (random walk) trend versus a deterministic trend, and to measure the persistence in the real GDP and GNP for the economy of Puerto Rico. It also attempted to measure the two main components of the real output variability: permanent versus the transitory component.

The presence of a stochastic trend was evaluated by the unit root test using the Dickey-Fuller approach and by the persistence measures such as the variance ratio test and the infinite sum of moving average coefficients (known also as the long-run response (LRR)). A test for shifting trends a la Perron was also considered due to the strong structural changes which occurred in Puerto Rico. The statistical test for unit root (Dickey-Fuller test) supports the presence of a random walk or a stochastic trend in either annual or quarterly figures of real GNP/GDP figures of Puerto Rico. Meanwhile, the variance ratio and other measures of persistence emphasized the presence of a large random walk component.

Using alternative ARIMA models, but under different specifications, the fraction of the permanent component attributed to the changes in real output was investigated. The results pointed towards a relatively significant proportion of the permanent component as a explanatory factor in real output variations in the short-run. This permanent fraction ranges from 14 percent to a about 60 percent.

Shifting or segmented trend was also investigated using Perron's procedures. Results tend to support the presence of a unit root or the presence of a stochastic trend even when exogenous shocks are included in the function, such as 1973-74 oil-price hikes. This is equivalent to say that real output in Puerto Rico does not exhibit any mean-reversion process;

rather, small and larger shocks, other than 1973-74 oil-price hikes, have affected real output behaviour in the short-run.

These results tend to support RBC hypothesis that economic time series, especially real output exhibits a stochastic trend rather than a deterministic one. Therefore, these time series are the results of particular shocks, not only arising from the transitory component but more important yet from the permanent component which is asserted mainly as an aggregate supply phenomenon.

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